A less than magic bullet

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1 Magic Bullet

Roy Lewallen in the *ARRL Antenna Book*[1] named a lossless network that would produce currents through two driven impedances with equal magnitude, and a relative 90 degree phase, independent of the values of the two driven impedances, a "magic bullet." If the magic bullet existed, it would make it possible to feed antenna elements in quadrature without having to make accurate measurements of the self and mutual impedances of the elements.

The general magic bullet would produce any desired relative phase. In the following I show that such a network cannot be realized with a reciprocal lossless network except for phase differences of 0 or 180 degrees. A nonreciprocal lossless network can produce other phase shifts. I show how to make a nonreciprocal transmission line by using a circulator. This produces the magic bullet, but the requirement that nonreciprocal elements are needed will probably limit the usefulness to UHF and microwave frequencies.

2 Three-Port Analysis

The system can be thought of as a general three-port network. Take port 1 to be the input port, and ports 2 and 3 to be output ports. Define the current to flow out of the network when the current is positive. The admittance parameters y_{ij} are defined by

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}. \tag{1}$$

For convenience, I take the driving voltage on port 1 to be 1. The impedances connected to ports 2 and 3 are Z_2 and Z_3, so that $V_2 = Z_2 I_2$ and $V_3 = Z_3 I_3$. Plugging these in to the admittance equation gives the equations

$$\begin{aligned} (1 - y_{22}Z_2)I_2 - y_{23}Z_3I_3 &= y_{21}, \\ -y_{32}Z_2I_2 + (1 - y_{33}Z_3)I_3 &= y_{31}. \end{aligned} \tag{2}$$

Solving for the ratio of the currents gives,

$$\frac{I_2}{I_3} = \frac{y_{21} - y_{33}y_{21}Z_3 + y_{23}y_{31}Z_3}{y_{31} - y_{22}y_{31}Z_2 + y_{32}y_{21}Z_2}. \tag{3}$$

This ratio must be constant independent of Z_2 and Z_3 for there to be a "magic bullet." This requires that the numerator and denominator be the same function of Z_2 and Z_3 except for an overall constant factor. Since the numerator has no Z_2 dependence, the coeficient of the Z_2 term in the denominator must be zero. Similarly since the denominator has no Z_3 dependence the coefficient of Z_3 in the numerator must be zero. The condition on the constant term for a magic bullet with a phase difference of ϕ radians is

$$\frac{y_{21}}{y_{31}} = \exp(j\phi). \tag{4}$$

Therefore y_{21} and y_{31} cannot be zero, and the requirements on the coefficients of Z_2 and Z_3 give:

$$y_{22} = y_{23} = y_{32} = y_{33} = 0. \tag{5}$$

For the cases of 0 and 180 degrees, Lewallen's current forcing solution using 1/4 wavelength lines, gives $y_{22} = y_{32} = y_{23} = y_{33} = 0$, and $y_{21}/y_{31} = \pm 1$, as required.

A lossless network cannot absorb power. This requirement is expressed by the equation

$$\sum_{m,n} V_m^*(y_{mn} + y_{nm}^*)V_n = 0, \tag{6}$$

and must hold for all driving voltages. If I terminate two of the ports with short circuits, and drive the third, I find that the diagonal elements of the y matrix must be purely imaginary. If I terminate one of the ports with a short circuit, and drive the other two at various relative phases, I get the requirement $y_{mn}^* = -y_{nm}$ for a lossless network.

If I now assume that the network is reciprocal, $y_{mn} = y_{nm}$. Therefore the off diagonal elements must be purely imaginary as well. The ratio of two of these admittances can have only a phase of 0 or 180 degrees.

This says that for a reciprocal lossless network, only 0 or 180 degree phase differences can be "current forced." Since capacitors, inductors, transformers, and normal transmission lines are reciprocal elements, no combination of them can produce the magic bullet.

If I introduce ferrites, or other nonreciprocal elements, it is possible to current force other phase differences.

The form of the admittance matrix required for current forcing is suggestive,

$$\begin{bmatrix} y_{11} & -y_{21}^* & -y_{31}^* \\ y_{21} & 0 & 0 \\ y_{31} & 0 & 0 \end{bmatrix}. \tag{7}$$

This form says that there cannot be any interaction between ports 2 and 3 except through their connection to port 1. That means that this form is synthesized by connecting two 2-port networks. One from port 1 to port 2, and the other from port 1 to port 3. These networks must have admittance matrices with a lossless "current forcing" form:

$$\begin{bmatrix} 0 & -\alpha^* \\ \alpha & 0 \end{bmatrix}. \tag{8}$$

3 Synthesis

Since the reciprocal case can be synthesized using a transmission line, I attempt to synthesize the nonreciprocal case using a lossless nonreciprocal transmission line. Such a line that can be physically realized has a characteristic impedance Z_0, with an electrical length for the forward mode of a radians, and b radians for the reverse mode. Its ABCD parameters are obtained by expanding in the modes:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1/Z_0 & -1/Z_0 \end{bmatrix} \begin{bmatrix} \exp(-ja) & 0 \\ 0 & \exp(jb) \end{bmatrix} \begin{bmatrix} 1 & Z_0 \\ 1 & -Z_0 \end{bmatrix} \tag{9}$$

$$= \frac{1}{2} \begin{bmatrix} \exp(-ja) + \exp(jb) & Z_0(\exp(-ja) - \exp(jb)) \\ (\exp(-ja) - \exp(jb))/Z_0 & \exp(-ja) + \exp(jb) \end{bmatrix}. \tag{10}$$

To have the current forcing form, we need to zero the diagonal terms. That requires $a + b$ to be an odd multiple of π, which then gives $\exp(jb) = -\exp(-ja)$. The result for the ABCD matrix is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & Z_0 \exp(-ja) \\ \exp(-ja)/Z_0 & 0 \end{bmatrix}. \tag{11}$$

These are lines that force a current of magnitude V/Z_0 with a phase lag of a. Notice that only for the case of $a = b$, both equal to an odd multiple of $\pi/2$ will the line be reciprocal. These reciprocal lines are Lewallen's odd multiple of a quarter wavelength lines.

The admittance matrix can be derived from the ABCD matrix to give:

$$\alpha = \exp(-ja)/Z_0 \tag{12}$$

A standard circulator can be used to produce current forcing lines with any phase shift. The general line can be produced using a circulator where power into port 1 goes to port 2, power into port 2 goes to port 3, and power into port 3 goes to port 1. If I put an adjustable shorted line on port 3, the forward wave travels “directly” to port 2, the reverse wave travels from 2 to 3 where it travels up the adjustable line, reflects, and then travels back to port 1. By adjusting the line, I can make the electrical length in the reverse direction anything I want. I now simply add extra line to port 2 so the electrical length from 1 to 2 is what I need, and adjust the shorted stub on 3 to give a total electrical length that is an odd multiple of a half wavelength.

The problem with this nonreciprocal technology is that it works well only at UHF or higher frequencies.

The special case of the nonreciprocal element with $a = 0$ and $b = \pi$ was postulated by Tellegen[2] in 1948. He called it a gyrator using a mechanical analogy where nonreciprocal elements correspond to spinning flywheels (i.e. gyroscopes). Within a few years, a ferrite gyrator and other nonreciprocal elements for microwave frequencies had been described[3]. If a medium or high frequency gyrator could be constructed, it would provide the necessary element to produce a quadrature feed without measuring element impedances.

For a quadrature feed, one antenna is connected to a quarter wavelength transmission line, and the other to a gyrator of the same characteristic impedance. The ends of the gyrator and the quarter wavelength line are connected together and driven by the same source.

References

[1] R.W. Lewallen, page 8-14 in *The ARRL Antenna Book*, 17th Edition, edited by R.D. Straw, (ARRL,Newington,1994).

[2] B.D.H. Tellegen, *The Gyrator, a New Electric Network Element*, Philips Research Reports **3**, 81 (1948).

[3] C.L. Hogan, *The Ferromagnetic Faraday Effect at Microwave Frequencies and Its Applications*, Reviews of Modern Physics **25**, 253 (1953).